Emotions, Bayesian Inference, and Financial Decision Making

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Abstract

This paper presents a model in which rational and emotional investors are compelled to make decisions under uncertainty in order to ensure their survival. Using a neuroeconomic setting, we show that, when different investor types fight for market capital, emotional traders tend not only to influence prices but also to have a much more developed adaptive mechanism than their rational peers, in spite of their apparently simplistic demand strategy and distorted revision of beliefs. Our results imply that prices in financial markets could be seen more accurately as a thermometer of the market mood and emotions rather than as simple informative signals as stated in traditional financial theory.

Keywords: Emotions, Judgement Under Uncertainty, Bayesian Inference, Behavioral Economics, Neuroeconomics, Decision-Making Process.

JEL classification: G1

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1   Introduction

In contrast to the dominant view of the negative influence of emotions (Smith (1759), Peter and Slovic (2000)), new research in neuroeconomics, and psychophysiology have highlighted the benefits emotions bring to decision making. The purpose of this paper is in line with these recent findings by showing the important role of emotions in financial decision making as an adaptive mechanism. Also, we are interested in the use of moods and emotions as an analytical toolbox employed to establish trading strategies which appears to be as good as (if not better than) purely rational ones, in order to ensure survival in competitive environments.

Emotions are important adaptive toolboxes in speeding up the decision-making process. Neuroscientists document the existence of a permanent interaction between the neural systems of both the thinking and the feeling parts of the human brain. Moreover, due to the higher speed of emotional responses to external stimuli compared to the reasoning responses, or to a sufficiently high intensity of emotions, in certain situations human actions can be developed without thinking (Bosman and van Winden (2005)).

Many terms have been used to indicate emotions. The term "feeling" is a synonym for emotion, albeit with a broader range. In the psychological literature the term "affect" is used to imply an even wider range of phenomena that is in some way connected to emotions, moods, disposition and preferences. There is a consensus to use the term "emotion" or "emotional episode" for states that last a limited amount of time, i.e. between few minutes and a few hours, and the term "mood" for an emotional state that usually last for hours, days or weeks, sometimes as a low intensity background (Oatley and Jenkins, 1996).

From a neurological point of view, Damasio (2000) defines emotions as a specific and consistent collection of physiological responses triggered by certain brain systems when the organism represents certain objects or situations. An important feature of emotions is that they represent not one single response but rather collections of responses. Moreover, an emotion is always varied and complex. Emotions can be induced in an unconscious manner and thus appear as unmotivated to the conscious self. The usual inducers of emotions are representations of objects or situations that can come either from outside or from inside an organism. The kinds of stimuli that can cause emotions tend to be systematically linked to a certain kind of emotion. Emotions generate both a behavior as a reaction to the inducing situation and a change in internal state which prepares the organism for that particular behavior.

In a controversial paper, Shiv, Loewenstein, Bechara and Damasio (Shiv et al. (2005)) found that the emotionally impaired are more willing to gamble for high stakes than non-impaired people. Even more surprising is the fact than people with brain damage generally make better financial decisions that people

with normal IQs.

According to Loewenstein and Lerner (2003), emotions enter into decision making in two ways: as expected emotions, i.e. by the prediction of emotional consequences of the decision, and as immediate emotions, i.e. by the change generated in the decision maker's expectations regarding the probability or desirability of future decision effects.

Rational decision making has always been associated with the concept of Bayesian inference which can be seen as the cornerstone of modern decision theory, given the importance in assisting agents to make rational decisions under uncertainty. In a nutshell, Bayes rules assist agents in amending prior beliefs by a signal in which new information is condensed.

In economics and more specifically in finance, the traditional theory relies on the assumption that investors are able to process the relevant information at their disposal and form unbiased probability judgments on the basis of the Bayes rule. Therefore, in the traditional framework, all market information is reflected in prices, which become fully informative[1]. Thus, every opportunity to make profits by forecasting future prices is ruled out.

Bayesian inference helps financial decision making when there is a need to update a probability estimate in the light of new evidence. Psychologists have wondered if the Bayes rule truly describes how people revise their beliefs. Following Birnbaum (2004), we can classify psychological opinions in three periods: an early period which supported the Bayesian rule as a rough descriptive model of how humans combine and update evidence; a second period dominated by Kahneman and Tversky's assertion that people do not use base rates or respond to differences in validity of sources of evidence; and a more recent period showing that people indeed rely on base rates and source credibility, but combine this information by means of an averaging model which is not consistent with the Bayes rule. The distinctive feature of the averaging model is that the directional impact of information depends on the relation between the new evidence and the current opinion (Birnbaum and Stegner (1979), Anderson (1981)).

For numerous years economics has relied on the fallacy that people apply rational calculation to economic decisions, ruling their life by economic models. Numerous empirical studies have emphasized the perpetuated existence of "not fully" or "quasi rational" investors, who employ a small number of simple and quick rules of thumb in order to make decisions under uncertainty. These rules are denoted in psychological terms as heuristics and proved to be useful in practice, especially when decisions have to be made under time constraints. However, they can sometimes lead to systematic mistakes (biases).

Within the last decade, a new paradigm which tries to integrate the classical financial theory with the behavioral perspective has been developed (Lo (2004)). This new paradigm is based on Darwin's theory

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[1]This is commonly known as the efficient market hypothesis (Samuelson (1969), Fama (1970)).

on the evolution of species and considers individuals as organisms that try to maximize the survival of their species. Thus, their behavior is not intrinsic and exogenous, but evolves by natural selection and is adapted to the particular environment. Particularly during the decision making process, individuals develop heuristics in order to maximize the efficiency of their responses to uncertainty. However, since the environment is constantly changing, they can observe behavioral biases given the maladaptation to new circumstances.

Our intention is to go one step further on the adaptive market hypothesis proposed by Lo (2004) by incorporating market microstructure features and the role of emotions in financial decision making. Accordingly, we design the ecology of the market as a population consisting of one market maker and three types of investors: rational investors, emotional investors and noise traders. These species differ in the way they interpret information and make decisions.

Concerning the information updating process, rational investors rely on Bayesian inference, while emotional investors under(over)weight the prior relative to the current information. In essence, emotional investors are quite young and self confident. Most of them have no formal education concerning financial markets. Thus, they do not analyze the market development using sophisticated tools, relying instead on their own intuition and experience in the market. The market maker fixes prices efficiently, in linear dependence on the observed total order flow.

Regarding the strategies employed in order to ascertain their demands, rational investors maximize their expected group-profits, emotional investors trade in accordance with their subjective evaluation of the returns, that relies on affective processes, and noise traders act randomly.

We believe that these three investor categories resemble better a real market, given that in reality we observe professional traders who dispose of sufficient resources and motivation in order to make decisions in a way approaching the rational type, as well as trades impelled by exogenous reasons, resembling the random actions of the noise traders in our model. Moreover, some market participants may speculate in reality on public information in an intuitive and affect-driven way.

Our paper shows that in the ecology of the market, not only rational, but also emotional investors can influence prices and survive in the long-run. Different types of market participants hold distinct beliefs on returns, which are directly revealed in their demand strategies and thus incorporated into prices, affecting the informational content of prices.

We test our model in an experimental environment showing that the emotional group's wealth can exhibit higher values than the rational one. This result supports the hypothesis concerning the survival (and dominance) of emotional traders in the market, in spite of their apparently simplistic strategy and "distorted" revision of beliefs.

The remainder of the paper is organized as follows: Section 2 presents the model of the ecology of the market including information-updating, pricing mechanism and demand strategies. The experimental design, encompassing the simulation results with respect to log-returns, group-demands and group-wealths is shown in Section 3. Section 4 summarizes the most important conclusions. Graphics and further results are included in Appendices A and B.

2 The model

This section shows how the subjective beliefs of different types of market participants are translated into prices. Our starting point is the modelling of the mind-set (or genesis of group-specific opinions) with respect to the evolution of market prices. Afterwards, we describe how investors' thoughts are translated into actions, that is how subjective opinions flow into idiosyncratic demand strategies. Then, we show how investor demands assemble the total order flow that periodically arrives to the market maker and how this order flow generates market prices. In other words, this section maps the entire process of price emergence, starting from its very first origin, the minds of market participants, going through their actions and reaching into market prices by means of the mechanisms employed by the market maker to fix prices. Thus, we can finally quantify the influence of different investor types on market prices.

We consider the population of market participants as consisting of *market maker* and *investors*. Investors take part in the trade for the purpose of obtaining profits. The market maker has as principal task to set up prices in order to maintain a fair, orderly, liquid and efficient market. To this end, she accommodates the buy and sell orders of investors and executes them at the quoted prices, so that she gains no profit from any of the buys and sells undertaken.

We consider the existence of three categories of investors active in the market: *rational investors*, *emotional investors* and *noise traders*. Rational investors act according to the traditional principles of Bayesian information updating and profit maximization. In contrast, emotional investors follow their intuition in evaluating the importance of different informational sources they access in order to revise their beliefs, as well as in translating these beliefs into periodical demands. Noise traders act randomly, being driven by exogenous reasons and their opinions do not influence the price evolution.

In the following subsections we present the information updating processes and the decisional mechanisms of the market participants and analyze their impact on price formation.

2.1 Information updating

We are primarily interested in how investors perceive information, which is subsequently incorporated in their trading strategies. Our focus is on the mental processes used by rational and emotional investors in

order to create and revise their trading strategies. Since noise traders act randomly, we are not interested in the way they perceive and update information.

Rational and emotional investors interpret the same market information in different ways. Formally, different beliefs are emphasized by distinct probability density functions[2], where $f^r(r_t|F_{t-1}) = f(r_t^r|F_{t-1})$ and $f^e(r_t|F_{t-1}) = f(r_t^e|F_{t-1})$ denote the densities of the rational and emotional expectations. In this context, $r_t = p_t - p_{t-1}$ stands for log-returns (and $p_t = \log(P_t)$ for log-prices), while F_{t-1} refers to the available market information at date t (consisting of past prices). Moreover, $r_t^r = E^r[r_t|F_{t-1}]$ and $r_t^e = E^e[r_t|F_{t-1}]$ denote rational and emotional subjective returns expectations, respectively.

Rational investors perform Bayesian updating, which means that their opinion revision relies on a "balanced" combination of prior and current information. In other words, they consider both information sources to be equally important in order to ascertain the current market price. In contrast, judgements of the emotional investors are affected by emotes.

Emotes represent units of emotions which serve to quantify emotional reactions. They occur in a short space of time and generate a cascade of feelings that develops into emotional habits, forcing investors to find a way to control them and/or to justify what is happening. These cognitive processes induced by emotions naturally interfere with logical reasoning, which was traditionally considered as the unique control factor in human decisions. Thus, the integration of emotions-driven processes in the formal decisional setting generates departures from the traditional notion of rationality. Acting in accordance with the current affective state entails making impulsive decisions, thus to the implementation of simple heuristics (rules of thumb) which, as cognitive psychologists have pointed out, can be helpful in certain decision situations (Gigerenzer et al. (1999)), whilst sometimes leading to systematic errors (Tversky and Kahneman (1974)).

Emotional investors concentrate on the affective reactions and use their explanatory power in order to make judgments and choices. In other words, emotional investors rely more in their intuition or feeling about past and new information.

Formally, rational investors combine prior and current information in a balanced way, while emotional investors can over(under)appreciate the importance of past information with respect to new evidence. The density functions of rational and emotional expectations $f^r(r_t|F_{t-1})$ and $f^e(r_t|F_{t-1})$ result from the combination of two elements: the distributions of log-returns conditional on the subjective investor expectations (which correspond to the current information) and the prior distribution of these subjective

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[2]Henceforth, the superscripts r and e emphasize the subjectivity of the rational and emotional view, respectively.

expectations (which corresponds to the prior information)[3]

$$f^r(r_t|F_{t-1}) = \int\limits_{-\infty}^{\infty} \int\limits_{-\infty}^{\infty} \int\limits_{-\infty}^{\infty} g^r(r_t|r_t^r, r_t^e, r_t^n, F_{t-1}) \varphi^r(r_t^r, r_t^e, r_t^n|F_{t-1}) dr_t^r dr_t^e dr_t^n \tag{1a}$$

$$f^e(r_t|F_{t-1}) = \int\limits_{-\infty}^{\infty} \int\limits_{-\infty}^{\infty} \int\limits_{-\infty}^{\infty} [g^e(r_t|r_t^r, r_t^e, r_t^n, F_{t-1})]^b [\varphi^e(r_t^r, r_t^e, r_t^n|F_{t-1})]^a dr_t^r dr_t^e dr_t^n, \tag{1b}$$

where a and b are chosen in order to satisfy

$$\int\limits_{-\infty}^{\infty} f^e(r_t|F_{t-}) dr_t = 1. \tag{2}$$

The power-weights a and b allow us to formally model the idea of affect-driven information updating, as employed by emotional investors. When $b \geq a > 0$, emotional investors react myopically to current market events, given that the affective reaction carried by new information prevails. In contrast, when $a \geq b > 0$, investors consider that new evidence is not as important as their subjective beliefs formed in the past. [4]

In order to derive the subjective distributions (1), we formulate the two different informational components that emerge from the conditional current information distributions (g) and the densities of group-specific prior expectations (φ).

Rational and emotional investors are aware that actions based on their subjective beliefs influence market prices. Therefore, the distributions g^r and g^e refers to the expected returns given that investors have already established their subjective opinions. We consider that, conditionally on the subjective expectations of the three investor types and the past information, rational investors expect log-returns to be normally distributed. The distribution g^r centers on a linear combination of subjective expectations originating from each investor type active in the market (r_t^r, r_t^e and r_t^n)[5] and contains an exogenous noise component. In other words, rational investors are aware of the existence of different investor types which hold different opinions about price evolution. In contrast, emotional investors focus on their own intuition and do not account for other opinions in the market. Thus, the distribution of current information in the view of emotional investors g^e centers only on their own affective expectation and encompasses the same

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[3]The emotional updating rule is defined in the style of Grether (1980) and Shefrin (2005). In spite of the similitude with the Bayesian formula, we note that emotional investors do not perform Bayesian updating in traditional sense, instead performing a kind of affective (adaptive) Bayesian updating inspired by averaging model of Birnbaum and Stegner (1979).

[4]The marginal cases of mathematical interest: $a = 0$ and $b = 0$ appear to be irrelevant from an economical point of view, given that either uninformative prior or uninformative current information is hardly to be expected in conjunction with real decision problems. Shefrin (2005) suggests as a more realistic assumption the following values: $b > 1$ and $0 < a < 1$.

[5]The notation r_t^n corresponding to a subjective noise trader expectation over log-returns is only formal. Noise traders' beliefs are not of interest for the price evolution and noise traders do not develop a well-defined demand strategy. This notation is meant to facilitate the understanding of price formation and the comparison between the behavior of different market participants.

exogenous noise as g^r

g^r :

$$r_t | r_t^r, r_t^e, r_t^n, F_{t-1} \sim N(c_{t-1} + c^r r_t^r + c^e r_t^e + c^n r_t^n, \sigma^2) \tag{3a}$$

g^e :

$$r_t | r_t^r, r_t^e, r_t^n, F_{t-1} \sim N(k_{t-1} + k^e r_t^e, \sigma^2). \tag{3b}$$

where c^r, c^e, c^n and k^r, k^e represent the weights of each group-specific expectations in the view of the rational and emotional investors. Also, c_{t-1} and k_{t-1} denote constant terms relying on the available (past) information.

The rational and emotional joint prior densities from equations (1) rely on the prior distributions of each subjective expectation given the past information. Rational investors consider the prior subjective expectations as independent. On the other hand, emotional investors are not concerned with the formation of expectations by other investors, so they treat the prior rational and the prior noise trader expectations as being uninformative. Formally, this leads to

$$\varphi^r(r_t^r, r_t^e, r_t^n | F_{t-1}) = \varphi^r(r_t^r | F_{t-1}) \varphi^r(r_t^e | F_{t-1}) \varphi^r(r_t^n | F_{t-1}) \tag{4a}$$

$$\varphi^e(r_t^r, r_t^e, r_t^n | F_{t-1}) = \varphi^e(r_t^e | F_{t-1}). \tag{4b}$$

Rational and emotional investors assign different normal distribution laws for the prior subjective densities from equations (4). We formulate them as follows

$$\varphi^r(r_t^r | F_{t-1}) : \qquad\qquad r_t^r | F_{t-1} \sim N(\tilde{r}_{t-1}^r, (\sigma^r)^2) \tag{5a}$$

$$\varphi^r(r_t^e | F_{t-1}) : \qquad\qquad r_t^e | F_{t-1} \sim N(\tilde{r}_{t-1}^e, (\sigma^e)^2) \tag{5b}$$

$$\varphi^r(r_t^n | F_{t-1}) : \qquad\qquad r_t^n | F_{t-1} \sim N(\tilde{r}_{t-1}^n, (\sigma^n)^2), \tag{5c}$$

for the rational investors and

$$\varphi^e(r_t^e | F_{t-1}) \quad : \quad r_t^e | F_{t-1} \quad \sim N(\tilde{r}_{t-1}^e, (\sigma^e)^2), \tag{6}$$

for the emotional investors, respectively.

Finally, the derivation of the subjective densities $f^r(r_t | F_t)$ and $f^e(r_t | F_t)$ takes place by incorporating equations (3)-(6) into equations (1),[6]

$$r_t^r | F_{t-1} \sim N(c_{t-1} + c^r \tilde{r}_{t-1}^r + c^e \tilde{r}_{t-1}^e + c^n \tilde{r}_{t-1}^n, \sigma^2 + (c^r \sigma^r)^2 + (c^e \sigma^e)^2 + (c^n \sigma^n)^2) \tag{7a}$$

$$r_t^e | F_{t-1} \sim N\left(k_{t-1} + k^e \tilde{r}_{t-1}^e, \frac{\sigma^2}{b} + \frac{(k^e \sigma^e)^2}{a}\right). \tag{7b}$$

We note there is a discrepancy between rational and emotional views over returns distribution, which is mainly driven by emotes. Rational investors allow for the existence of different opinions in the market

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[6] According to equations (1), equations (7) represent marginal distributions resulting from the combination of prior and current information. Hence, they are different of the prior distributions (5) and (6).

and combine past and new information in a balanced way. In contrast, emotional investors guide their beliefs by means of their current affective state, which leads to a focussing on the own price expectation and an impulsive over(under)weighting of one of the informational sources considered in order to revise beliefs. Thus, the behavioral power weights a and b have an impact on the variance of the emotionally perceived returns.

As already mentioned, rational investors recognize the existence of other investor categories in the market. In principle, they are aware of the fact that some investors trade randomly and others follow their intuition. Furthermore, the noise component of both emotional and noise traders prior beliefs is considered to be identical (being generally denoted as σ^η). However, this does not make emotional investors act as pure noise traders. Relying on their intuition, emotional investors add a deterministic component to their expectations (and thus demands), as in equation (5b), which is different from $\tilde{r}^n_{t-1} = 0$. Formally, we render these assumptions as

$$\tilde{r}^n_{t-1} = 0 \tag{8a}$$

$$\sigma^e = \sigma^n = \sigma^\eta. \tag{8b}$$

2.2 Pricing Mechanism

In order to describe the demand strategies, we first formulate the law used by the market maker for setting the current price. Relying on commonly accepted market microstructure approaches (i.e. Kyle (1985), Farmer (2002)), we assume that the market maker fixes prices efficiently, in linear dependence on the total flow of market orders issued by the investors

$$r_t \quad = r_0 + \lambda Q_t, \tag{9}$$

where λ represents the inverse market liquidity and Q_t is the total order flow observed by the market maker at date t. Henceforth, we consider zero initial returns $r_0 = 0$.

The total order flow represents the sum of market orders currently issued by investors

$$Q_t \quad = n^r q^r_t + n^e q^e_t + (1 - n^r - n^e) q^n_t, \tag{10}$$

with n^r and n^e being the relative dimensions of the rational and emotional group, respectively.[7] q^r_t, q^e_t and q^n_t represent the demands of the rational investors, emotional investors and noise traders.

2.3 Asset demands

The formation and revision of subjective beliefs described in Subsection 2.1 directly influences the actions of the investors. These actions center on investment decisions which are periodically undertaken and

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[7]Henceforth, we consider $n^r, n^e \neq 0$.

followed by each group-specific logic of concrete strategies of asset demand. This subsection describes the demand strategies of each investor type active in the market, which generate the total order flow arriving at the market maker, as formulated in equation (10).

Rational decisions are dominated by reasoning (which complies with the complex of cognitive processes denoted as System 2 within the Kahneman's (2003) two-system model). Rational investors act in accordance to the traditional principles of expected profit maximization. The trading strategy of the pure noise traders is mostly driven by exogenous reasons (such as the need of liquidity) and therefore based on purely random actions. Emotional investors care about their affective reactions and intuitions in devising their demand. From a psychological point of view, their cognitive processes are dominated by affect (Loewenstein and Lerner (2003)) and intuition (denoted as System 1 in Kahneman (2003)).

In light of evolutionary evidence about group adaptation to the environment, we consider investors to be mainly concerned not only with individual survival, but also with the survival of their own kind.[8] Thus, rational investors aim the maximization of their subjectively expected current group-profit, which can be formulated as a product of the current demand and the difference between subjective and market expected log-prices

$$q_t^r = \arg\max_{q_t}\{n^r q_t (E^r[P_t|F_{t-1}] - E[P_t|F_{t-1}])\}. \tag{11}$$

Here, the subjective price expectation $P_t^r = \exp(p_t^r) = E^r[P_t|F_{t-1}]$ emerges from the idiosyncratic interpretation of public information byrational investors (as described in Subsection 2.1), while the market expectation $E[P_t|F_{t-1}]$ applies directly to market prices (as fixed by the market maker). In other words, the actual rational strategy accounts for the gap between what is subjectively considered to be the "true" price of the trade object and the market value of the asset.

Conditional on past information at date t, we can express the above difference as

$$E^r[P_t|F_{t-1}] - E[P_t|F_{t-1}] = E^r[P_t - P_{t-1}|F_{t-1}] - E[P_t - P_{t-1}|F_{t-1}]$$
$$\simeq P_{t-1}(r_t^r - E[r_t|F_{t-1}]).$$

Hence, the rational demand results in

$$q_t^r = \arg\max_{q_t}\{n^r q_t P_{t-1}(r_t^r - E[r_t|F_{t-1}])\}.$$

In a traditional setting (e.g. with sequential auction equilibrium as in Kyle (1985)), the market maker considers that only rational investors are smart enough in order to predict and influence prices. Therefore, the "objective" expectation on returns given the pricing mechanism (9) results in $E[r_t|F_{t-1}]) = \lambda n^r q_t^r$.

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[8]Given that we consider the group dimensions n^r and n^e as constant in time, there is no formal difference between the maximization of individual profits and of group-profits.

Hence, we derive

$$q_t^r = \arg\max_{q_t}\{n^r q_t P_{t-1}(r_t^r - \lambda n^r q_t)\} = \frac{1}{2\lambda n^r} r_t^r.$$

Defining

$$\beta^r = \frac{1}{2\lambda n^r}, \tag{12}$$

the first and second order maximization conditions reduce to[9]

$$q_t^r = \beta^r r_t^r \qquad\qquad \Leftrightarrow q_t^r = \beta^r(p_t^r - p_{t-1}) \tag{13a}$$

$$\lambda > 0. \tag{13b}$$

β^r stays for the sensitivity of the rational demand to price movements, which is proportional to the market liquidity and inversely proportional to the dimension of the rational group. Therefore, a more liquid market gives rise to an increase in the rational demand. This corresponds to the empirical finding that rational strategies are more successful in liquid markets.

Equation (13a) shows that, acting as maximizers of the subjectively perceived group-profits, rational investors get around using an ordered-based value strategy (as in Farmer (2002)). This strategy aims to linearly exploit the difference between subjectively expected log-prices and the log-price of the last period. The intuition behind this strategy can be traced back to the natural tendency to buy, as long as assets appear to be low priced relative to their own speculation, and to sell otherwise.

According to equation (7a) and (8), rational returns expectation can be formulated as

$$r_t^r = c_{t-1} + c^r \tilde{r}_{t-1}^r + c^e \tilde{r}_{t-1}^e + \zeta_t + c^r \epsilon_t + (c^e + c^n)\eta_t, \tag{14}$$

where

$$\zeta_t \sim \text{ iid. } N(0, \sigma^2) \tag{15a}$$

$$\epsilon_t \sim \text{ iid. } N(0, (\sigma^\epsilon)^2) \tag{15b}$$

$$\eta_t \sim \text{ iid. } N(0, (\sigma^\eta)^2) \tag{15c}$$

$$\langle \zeta_t, \epsilon_t \rangle = \langle \zeta_t, \eta_t \rangle = \langle \epsilon_t, \eta_t \rangle = 0, \tag{15d}$$

and $\sigma^\epsilon = \sigma^r$.

Hence, the rational demand from equation (13a) results in

$$q_t^r = \beta^r c_{t-1} + \beta^r c^r \tilde{r}_{t-1}^r + \beta^r c^e \tilde{r}_{t-1}^e + \beta^r \zeta_t + \beta^r c^r \epsilon_t + \beta^r (c^e + c^n)\eta_t \tag{16a}$$

$$q_t^r | F_{t-1} \sim N(\beta^r c_{t-1} + \beta^r c^r \tilde{r}_{t-1}^r + \beta^r c^e \tilde{r}_{t-1}^e, (\beta^r \sigma)^2 + (\beta^r c^r \sigma^\epsilon)^2 + [\beta^r (c^e + c^n)\sigma^\eta]^2). \tag{16b}$$

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[9]Henceforth, we exclude the hypothetical limit case of a perfect liquid market $\lambda = 0$.

Thus, the rational demand centers on a combination between past rational and emotional expectations and accounts for all types of noise in the market: exogenous noise (ζ), rational noise (ϵ) and noise trader noise (η).

Emotional investors follow their intuition in formulating demands. They simply react proportionally to their current expectation of returns

$$q_t^e \quad = \beta^e r_t^e, \tag{17}$$

where β^e describes the sensitivity of emotional demand to price changes and $r_t^e = E^e[r_t|F_{t-1}]$ is the emotional expectation over log-returns conditional on public information.[10]

According to equation (7b), the emotional returns expectation can be formulated as

$$r_t^e \quad = k_{t-1} + k^e \tilde{r}_{t-1}^e + \frac{1}{\sqrt{b}} \zeta_t + \frac{k^e}{\sqrt{a}} \eta_t, \tag{18}$$

where ζ_t and η_t are distributed according to equations (15).

Even if the emotional strategy contains a noise component similar to the noise trader strategy η_t, it additionally accounts for exogenous shocks and depends on the behavioral information updating parameters a and b

$$q_t^e = \beta^e k_{t-1} + \beta^e k^e \tilde{r}_{t-1}^e + \frac{\beta^e}{\sqrt{b}} \zeta_t + \frac{\beta^e k^e}{\sqrt{a}} \eta_t \tag{19a}$$

$$q_t^e|F_{t-1} \sim N\left(\beta^e k_{t-1} + \beta^e k^e \tilde{r}_{t-1}^e, \frac{(\beta^e \sigma)^2}{b} + \frac{(\beta^e k^e \sigma^\eta)^2}{a}\right). \tag{19b}$$

Emotional investors can use differences in past price levels as a proxy for $\tilde{r}_{t-1}^e$.[11] The emotional strategy centers on emotional expectations and accounts only for exogenous and noise trader noise.

Noise traders act randomly

$$q_t^n = r_t^n = \eta_t \tag{20a}$$

$$q_t^n|F_{t-1} \sim N(0, (\sigma^\eta)^2). \tag{20b}$$

Therefore, the total order flow issued by investors at date t according to equation (10) results in a linear combination of subjective expectations over log-returns

$$\begin{aligned} Q_t \quad &= n^r \beta^r c_{t-1} + n^e \beta^e k_{t-1} + n^r \beta^r c^r \tilde{r}_{t-1}^r + (n^r \beta^r c^e + n^e \beta^e k^e) \tilde{r}_{t-1}^e \\ &+ \left(n^r \beta^r + \frac{n^e \beta^e}{\sqrt{b}}\right) \zeta_t + n^r \beta^r c^r \epsilon_t + \left[n^r \beta^r (c^e + c^n) + \frac{n^e \beta^e k^e}{\sqrt{a}} + (1 - n^r - n^e)\right] \eta_t. \end{aligned} \tag{21}$$

---

[10] Please note that the mathematical equivalence of the rational and emotional strategies, according to equations (13a) and (17), is not based on a logical similarity in thinking. Rational traders aim the maximization of expected group-profit and their strategy *results* to be proportional to their own current returns expectation. Emotional traders simply follow their beliefs over current returns.

[11] Such as a trend-following strategy, as in Farmer (2002). E.g. $\tilde{r}_{t-1}^e = r_{t-1} - r_{t-2} = p_{t-1} - 2p_{t-2} + p_{t-3}$. Thus, they act in accordance with the idea that past prices do encompass enough information in order to provide a basis for investment decisions.

According to equation (9), the market maker fixes prices proportionally to the observed total order flow Q_t

$$
\begin{aligned}
r_t \quad &= \frac{c_{t-1}}{2} + \lambda n^e \beta^e k_{t-1} + \frac{c^r}{2} \tilde{r}_{t-1}^r + \left(\frac{c^e}{2} + \lambda n^e \beta^e k^e \right) \tilde{r}_{t-1}^e \\
&+ \left(\frac{1}{2} + \frac{\lambda n^e \beta^e}{\sqrt{b}} \right) \zeta_t + \frac{c^r}{2} \epsilon_t + \left[\frac{c^e + c^n}{2} + \frac{\lambda n^e \beta^e k^e}{\sqrt{a}} + \lambda (1 - n^r - n^e) \right] \eta_t.
\end{aligned}
\tag{22}
$$

Thus, the log-returns form as a linear combination of the subjective expectations of each investor type of current returns. Moreover, log-returns encompass a threefold noise, originating from the noise in rational, emotional and noise trader expectations, as well as from possible exogenous factors.

Equation (22) emphasizes the fact that both rational and emotional investors exert influence on prices. While the weight associated with the rational effect $\frac{c^r}{2}$ is constant, the emotional influence exhibits lower values in a more liquid market (i.e. for a higher λ), but remains bounded by the constant $\frac{c^e}{2}$.

When $\lambda n^e \beta^e k^e = -\frac{1}{2} c^e$, the direct impact of the emotional expectation on mean returns disappears, but emotional strategy continues to affect prices by means of other parameters, such as a, b, k^e and β^e.

Similarly, if rational investors are smart enough in order to infer the current emotional expectation, as well as the emotional demand sensitivity to price movements and the emotional constant k^e, they can compensate on average for the existence of emotional traders. Thus, with $c^r \tilde{r}_{t-1}^r = -(c^e + 2\lambda n^e \beta^e k^e) \tilde{r}_{t-1}^e$, emotional expectations have no direct influence on the mean current returns (but the emotional influence maintains in the constant part of the mean returns and in the returns variance). However, such a situation is economically improbable, given the amount of information the rational investors are assumed to be able to assess.

Moreover, equation (22) points out the fact that the pattern of the serial correlations of log-prices as well as of the cross-correlations of log-returns with the subjective expectations of different investor groups depends on both rational and emotional mean expectations $\tilde{r}_{t-1}^r$ and $\tilde{r}_{t-1}^e$, respectively. Here, the cross-correlations point out the ability of investors to infer valuable information about future returns from their own expectations. Further calculations and results for different particular cases with respect to the serial and cross-correlations are included in Appendix A.

3   Experimental Design

In this section, our intention is to show by means of simulation techniques the evolutionary dynamics of returns and rational and emotional investors' wealth by following the decision rules stated above. The analysis of the wealth evolution in time helps us to draw a final conclusion concerning the survival of the fittest.

We simulate series of $T = 1000$ log-returns r_t, according to equation (22). In doing so, we consider

different values for the model parameters. Firstly, we fix the proportion of noise traders at $1 - n^r - n^e = 0.05$ and consider three different values for the dimension of the emotional investors' group $n^e \in \{0.25, 0.5, 0.75\}$. Moreover, we consider the following standard deviations for the different exogenous and endogenous noise components of returns: $\sigma = 0.02$, $\sigma^\epsilon = 0.02$, $\sigma^\eta = 0.03$. In line with the results of Hasbrouck (2005), the inverse market liquidity parameter is taken as $\lambda = 0.08$. β^r is derived from equation (12) and the emotional demand sensitivity is considered to be in the same order of magnitude $\beta^e = \beta^r$. Furthermore, the parameters of the information updating processes are set to the following values $c_{t-1} = k_{t-1} = 0$, $c^r = n^r$, $c^e = n^e$, $c^n = 1 - n^r - n^e$, and $k^e = n^e$. The informational weights assessed by emotional investors to the prior and current information can also vary $a, b \in \{1, 0.01, 5\}$. Following Kyle (1985), we consider the rational mean expectations $\tilde{r}^r_{t-1} = -p_{t-1}$.[12] The emotional traders are assumed to pursue a trend-following strategy, which reduces to $\tilde{r}^e_{t-1} = r_{t-1} - r_{t-2}$.

We analyze the following cases:

1. Case 1: low proportion of emotional investors active in the market $n^e = 0.25$,

2. Case 2: medium proportion of emotional investors active in the market $n^e = 0.5$,

3. Case 3: high proportion of emotional investors active in the market $n^e = 0.75$.

For each of them, we consider three scenarios:

- Scenario a: emotional investors weight prior and current information in a balanced way $a = b = 1$,

- Scenario b: emotional investors underweight prior information $a = 0.01$ and overweight current information $b = 5$,

- Scenario c: emotional investors overweight prior information $a = 5$ and underweight current information $b = 0.01$.

Using the simulated returns series, we derive the group-specific asset demands q^r_t, q^e_t and q^n_t, according to equations (16a), (19a) and (21a), respectively, for each of the cases and scenarios considered.

Subsequently, we focus on group-wealth. At every date t, the wealth is given by the amount of risky asset units held by each investor group $(n^r q^r_t, n^e q^e_t$ and $(1 - n^r - n^e)q^n_t)$ valuated according to the change

---

[12]Kyle (1985) demonstrates the existence of an unique linear sequential auction equilibrium in a market setting with informed investors and noise traders. The informed investors are the counterpart of our rational traders. They are considered to be able to infer the so called "true" value of the traded asset v_t and to exploit the difference between it and the current price in their strategies. Thus, the rational strategy in our setting corresponds in Kyle's model to $q^r_t = \beta(v_t - p_{t-1})$, where $v_t = p_0 + \epsilon_t$ and $\epsilon_t \sim$ iid. $N(0, (\sigma^\epsilon)^2)$. Given that, according to equation (13a), the rational demand is proportional to the rational expectations, this yields to $\tilde{r}^r_{t-1} = E[v_t | \mathcal{F}_{t-1}] - p_{t-1} = p_0 - p_{t-1} = -p_{t-1}$.

in current price

$$W_t^r = W_{t-1}^r + n^r q_t^r (P_t - P_{t-1}) = W_0^r + n^r \beta^r \sum_{s=1}^{t} r_s^r [\exp(p_s) - \exp(p_{s-1})] \qquad (23a)$$

$$W_t^e = W_{t-1}^e + n^e q_t^e (P_t - P_{t-1}) = W_0^e + n^e \beta^e \sum_{s=1}^{t} r_s^e [\exp(p_s) - \exp(p_{s-1})] \qquad (23b)$$

$$W_t^n = W_{t-1}^n + (1 - n^r - n^e) q_t^n (P_t - P_{t-1})$$
$$= W_0^n + (1 - n^r - n^e) \sum_{s=1}^{t} r_s^n [\exp(p_s) - \exp(p_{s-1})], \qquad (23c)$$

where $P_t = \exp(p_t)$ represents the current price. It can be derived from current log-returns in virtue of equation (22).

3.1   Main results

This subsection presents the main results obtained for each case considered in the context of our simulations.

1. Case 1: low proportion of emotional investors active in the market $n^e = 0.25$

   Figures 1-3 from Appendix B illustrate the log-returns, group-demands and investor group-wealths for each scenario of case 1.

   The simulated returns exhibit similar means for all subcases considered within case 1. However, they become more volatile with the use of adaptive beliefs updating by emotional investors (i.e. under scenarios b and c), especially in case 1c, when emotional investors overweight prior and underweight current information.

   For each of the considered scenarios, the mean and variance of rational and emotional group-demands are of the same order of magnitude but always of contrary sign. In situations 1a and 1c, the mean rational group-demand is positive and the mean emotional group-demand negative, which points out that, on average, rational investors buy more, while emotional ones sell more. In contrast, case 1b (when emotional investors overweight prior information) yields, on average, positive values for the emotional group-demand and negative ones for the rational group-demand. Again, group-demands become more volatile under scenarios 1b and 1c.

   The most interesting pattern is shown by group-wealths. When emotional investors overweight current and underweight prior information in forming their beliefs (case 1b), they succeed in gaining more than rational investors in the beginning and in the last half of the simulated trade period. When emotional information updating develops similar to the Bayesian updating, the rational investors are better off and gain in average(case 1a). Rational investors continue to earn more from

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their trades also when the adaptive emotional beliefs formation consists in underweighting current and overweighting prior information (case 1c). However, in case 1c, emotional investors appear to recover in the end of the simulated series and their wealth is positive in average.

Comparing the results for each scenario of case 1, we observe that the average values of group-demands and group-wealths increase for both rational and emotional investors when the emotional information updating is adaptive. The highest mean values are reached under scenario c, at the price of more volatile returns. Thus, if adaptive information updating renders the market to be more unpredictable, it increases in exchange the chances of higher trade profits.

2. Case 2: medium proportion of emotional investors active in the market $n^e = 0.5$

   Simulated log-returns, as well as the demands and wealths of each investor group for all scenarios of case 2 are shown in Figures 4-6 from Appendix B.

   An increase in the proportion of emotional investors (up to values comparable with the proportion of the rational investors) leads to a stronger emotional influence on prices. This increases not only the survival chances of the emotional investors, but also yields an improvement of the general trade benefits, given that average wealth values are always positive and higher for all investor types compared to case 1. However, the market also becomes more volatile. The use of adaptive information updating results in a increased volatility of returns and demands.

   The emotional group-wealth now tracks the rational one more closely and emotional investors become the fittest in the market in the second half of the simulated trade period for both cases 2b and 2c, i.e. when they actually employ the affective beliefs updating techniques as described in Section 2.1 of our paper.[13]

   The mean rational and emotional group-demands maintain contrary signs for situations 2b (rational investors sell and emotional ones buy on average) and 2c (rational investors buy and emotional ones sell on average), but exhibit close values (small and positive) for case 2a. Similarly, in cases 2b and 2c, the variance of emotional group-demand is higher than the rational one. By contrast, situation 2a results in a rational demand twice as variable as the emotional one.

3. Case 3: high proportion of emotional investors active in the market $n^e = 0.75$

   For the considered values of the model parameters, the predominance of emotional investors in the market generates an explosion in prices after approximatively 50 trades. This relies on the high value of the emotional demand sensitivity we employ in our simulations, which, amplified by the

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[13]Further simulations for different values of the model parameters come to support the finding that such a situation does not represent an accident, but can naturally occur in real market settings.

high proportion of emotional traders, yields to an excessive emotional demand and destabilizes the market. At first sight, an excessively intense emotional activity appears to constitute a menace for a proper market functioning. However, a closer scrutiny of the emotional demand sensitivity to price movements in practice is required in order to draw more precise conclusions in this context.

We have also analyzed the difference between real and subjectively expected group-incomes of rational and emotional investors. The real incomes are calculated at each date t as the product between current demand and changes in real prices (i.e. $n^r q_t^r (P_t - P_{t-1})$ and $n^e q_t^e (P_t - P_{t-1})$, respectively). The subjectively expected incomes rely on subjective rational and emotional returns expectations from equations (14) and (18), respectively (i.e. $n^r q_t^r (P_t^r - P_{t-1}^r)$ and $n^e q_t^e (P_t^e - P_{t-1}^e)$). Hence, differences between real and expected periodic incomes originate in the discrepancy between actual market prices and subjective beliefs of the rational and emotional investors $P_t - P_t^r$ and $P_t - P_t^e$, respectively.

We note that, on average, both rational and emotional differences in real and expected incomes, as well as the variances of these differences are small. In cases 1a, 1b and 2a, rational investors appear to overestimate real prices on average, which yields negative mean discrepancies between real and perceived periodic group-incomes. In most of the other considered cases, both rational and emotional investors exhibit comparable perceptions, by underestimating real prices on average.

3.2   Further results

In addition to the main results presented above, we replicate the simulations for a more liquid market, considering $\lambda = 0.001$. While the parameter β^r changes according to equation (12), we keep the same values for β^e as in the first series of simulations. With a lower inverse liquidity λ, β^r increases, which results in an increased rational demand and a higher rational influence on prices. Given that the difference between β^r and β^e is now of two orders of magnitude, rational investors dominate the market.

In general, returns in the more liquid market setting are less volatile and more stable, while group-demands and group-wealths exhibit larger average values than in the less liquid market setting analyzed in Subsection 3.1. Moreover, prices do no longer explode when emotional investors are present in excessive extent in the market (case 3).

Group-demands of rational and emotional investors preserve the general pattern shown in the less liquid market setting, being in most of the cases of contrary sign on average (when rational investors buy, emotional ones sell). The average amounts of shares traded by rational investors are higher for case 1, but becomes comparable to the trades of the emotional investors with an increase in the proportion of emotional investors active in the market.

For all considered cases, scenario b, as well as case 3a, entail positive mean values for emotional group-

wealths. Therefore, when emotional investors overweight current and underweight prior information, they gain on average (even when the market is more liquid). It is interesting to show that emotional investors make highest profits in case 3b, when they succeed in gaining profits close to the rational ones at the end of the trade period.

We report the correspondent log-returns, group-demands and group-wealths evolutions for all considered scenarios in Figures 7-9 from Appendix B.[14]

Subsequently, we try include in our simulations the fact that rational and emotional investors cannot sustain high losses for a prolonged period. When one of these two investor types gets out of the market, its group-demand as well as its group-wealth reduce permanently to zero and the market continues to function only with the rest of investors active. We assume that, given the randomness of their demand, noise traders as a group always remain active in the market.

Considering the same parameter values as in the simulations from Subsection 3.1, we generate another $T = 1000$ log-returns r_t. We allow as maximum a loss of -5 monetary units and a maximum loss duration of 50 or 100 trades.

The results show that emotional investors get out of the market in case 1 under all scenarios, as well as in case 2a. We find this result not very surprising, given that the excessive presence of rational investors in the market (as in case 1) increases their influence on prices and leaves lower survival chances for emotional investors. Moreover, under scenario a, emotional investors do not use an adaptive thinking and their strategy is poorer than that of their rational peers. However, rational investors are also forced out of the market in case 1b in the end of the simulated trade period. None of the investor groups runs out of money in the remaining situations.

In sum, the simulations emphasize the fact that possible situations (i.e. phases in the market evolution) exist, where emotional investors are better off then the rational ones. Their chances of survival and success increase with the use of affective information updating techniques. This is a clear evidence that, under certain circumstances, emotional investors have high chances of continued existence, which contradicts the traditional conviction that rational traders are the sole survivors.

4   Conclusions

Our paper aims to analyze the role of emotions in financial decision making. To this end, we model a market where different types of market participants trade a unique risky asset. These market participants are a market maker and three distinct investor groups: rational investors, emotional investors and noise

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[14]Due to the higher average values obtained for group-demands and group-wealths relative to the less liquid market setting from Subsection 3.1m we use different scales for the correspondent figures.

traders. We ascertain the formation and updating of individual beliefs, the investor demand strategies, their group-wealths, as well as the price fixation process undertaken by the market maker.

What makes the distinction between the investor types is the way they interpret public information in order to form and update their subjective returns expectations, as well as the strategy they pursue in order to determine the optimal trading volume. Rational investors form expectations by combining past and current information in a traditional Bayesian manner, and maximize the expected group-profits in formulating periodical asset demands. In contrast, emotional investors update information in an adaptive manner, putting different weights on distinct information sources. They also form their demands in an impulsive way by following their own expectation over returns. Noise traders act randomly.

The formation and revision of beliefs draws upon information updating. Our paper concentrates on the information updating processes of rational and emotional investors. We focus on the role of emotional investors as a distinct type of traders driven by affect and intuition. We suggest a way to quantify the emotional process of belief revision, showing how emotional investors may balance between past information and new evidence in contrast to the traditional Bayesian updating employed by rational investors.

Beliefs flow into subjective expectations of returns. The demand strategies of the investors are shaped in linear dependence on these subjective returns expectations. We show that the distinct beliefs of different market participants is directly reflected in the informational content of prices.

Furthermore, we test our model in an experimental environment for different values of the model parameters and in different market settings. Thus, we examine the evolution of prices, group-demands and group-wealths for various proportions of rational and emotional investors active in the market, as well as for two different values of market liquidity. We also analyze the survival of rational and emotional investors considering a market setting where they cannot sustain high long-lasting losses.

Our results show that there are situations where emotional investors appear to gain more money than the rational ones. This finding remains as evidence for the possible survival (and even dominance) of emotional traders in the market, in spite of their apparent simplistic strategy and non-traditionalist revision of beliefs. This contradicts the traditional conviction that only rational investors can survive in the long run. Emotional investors improve their chances of survival with the use of adaptive information updating techniques as modeled in our paper. Similarly, an increase in the proportion of emotional investors active in the market provides long-run benefits not only for themselves as a group, but also for other market participants.

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5 Appendix A

In order to derive the price and return serial correlations, we consider three particular cases:

1. no emotional influence in mean returns: $2\lambda n^e \beta^e k^e + c^e = 0$ and rational strategy following Kyle (1985): $\tilde{r}^r_{t-1} = -p_{t-1}$

   According to equation (22), log-returns yield to

$$
\begin{aligned}
r_t \quad = & \frac{c_{t-1}}{2} - \frac{c^e k_{t-1}}{2k^e} - \frac{c^r}{2}p_{t-1} + \frac{1}{2}\left(1 - \frac{c^e}{k^e \sqrt{b}}\right)\zeta_t + \frac{c^r}{2}\epsilon_t \\
& + \left[\frac{c^e + c^n}{2} - \frac{c^e}{2\sqrt{a}} + \lambda(1 - n^r - n^e)\right]\eta_t.
\end{aligned}
\tag{24}
$$

   With

$$
\gamma = \frac{c_{t-1}}{2} - \frac{c^e k_{t-1}}{2k^e} \tag{25a}
$$

$$
\gamma^r = \frac{c^r}{2} \tag{25b}
$$

$$
\xi_t = \frac{1}{2}\left(1 - \frac{c^e}{k^e \sqrt{b}}\right)\zeta_t + \frac{c^r}{2}\epsilon_t + \left[\frac{c^e + c^n}{2} - \frac{c^e}{2\sqrt{a}} + \lambda(1 - n^r - n^e)\right]\eta_t \tag{25c}
$$

$$
\tilde{\sigma}^2 = \left[\frac{1}{2}\left(1 - \frac{c^e}{k^e \sqrt{b}}\right)\sigma\right]^2 + \left[\frac{c^r}{2}\sigma^\epsilon\right]^2 + \left[\left(\frac{c^e + c^n}{2} - \frac{c^e}{2\sqrt{a}} + \lambda(1 - n^r - n^e)\right)\sigma^\eta\right]^2, \tag{25d}
$$

   log-prices reduces to

$$
p_t \quad = \gamma + (1 - \gamma^r)p_{t-1} + \xi_t. \tag{26}
$$

   If

$$
0 \quad < \gamma^r = \frac{c^r}{2} < 2 \tag{27}
$$

   the log-prices are stationary and the serial correlations result in

$$
\rho^p_0 = \frac{1}{\gamma^r(2 - \gamma^r)}\tilde{\sigma}^2 \tag{28a}
$$

$$
\rho^p_i = (1 - \gamma^r)^i \text{, for i} \geq 1. \tag{28b}
$$

   When $0 < \gamma^r < 1$, the log-price serial correlations are always positive. When $1 < \gamma^r < 2$, the first log-price serial correlation is always negative and the subsequent serial correlations exhibit contrary sign. Thus, log-prices turn to be mean-reverting.

   Expression (31) shows that trading in our market setting reduces the rational noise encompassed in σ^ϵ if $c^r < \frac{1 + \sqrt{13}}{2}$, due to the fact that the log-prices volatility is lower than the rational noise volatility $\tilde{\sigma} < \sigma^\epsilon$. Moreover, for $c^r = n^r$ and $c^e = k^e = n^e$, the exogenous noise comprised in σ is

also reduced. In this case, given that we expect the inverse market liquidity $\lambda < 1$, prices do not amplify the noise from σ^η.

From equation (26), log-returns follow a moving average

$$r_t \quad = \gamma - \gamma^r r_{t-1} - \gamma^r r_{t-2} - ... - \gamma^r r_1 + \xi_t. \tag{29}$$

The conditions for the stationarity of log-returns result from the correspondent characteristic equation. If they are fulfilled, then log-return serial correlations entail to

$$\rho_0^r = \frac{2}{2 - \gamma^r} \tilde{\sigma}^2 \tag{30a}$$

$$\rho_i^r = -\frac{\gamma^r (1 - \gamma^r)^{i-1}}{2} \ , \text{ for i} \geq 1. \tag{30b}$$

For $0 < \gamma^r < 1$ the log-return serial correlations are always negative. Otherwise they exhibit contrary signs.

2. rational strategy following Kyle (1985): $\tilde{r}_{t-1}^r = -p_{t-1}$ and emotional strategy: $\tilde{r}_{t-1}^e = r_{t-1}$

   Using the same notations as in equations (25) and $\gamma^e = \frac{c^e}{2} + \lambda n^e \beta^e k^e$, the log-prices render to

$$p_t \quad = \gamma + (1 - \gamma^r + \gamma^e)p_{t-1} - \gamma^e p_{t-2} + \xi_t. \tag{31}$$

If

$$\gamma^r \neq 0 \tag{32a}$$

$$\gamma^e \neq 1 \tag{32b}$$

$$\gamma^r - 2\gamma^e \neq 2 \tag{32c}$$

$$0 < \frac{1 + \gamma^e}{\gamma^r (1 - \gamma^e)(2 - \gamma^r + 2\gamma^e)} \tag{32d}$$

$$0 < (\gamma^e)^2 + \gamma^r - \gamma^e \tag{32e}$$

the log-prices are stationary and the correspondent serial correlations render to

$$\rho_0^p = \frac{1 + \gamma^e}{\gamma^r (1 - \gamma^e)(2 - \gamma^r + 2\gamma^e)} \tilde{\sigma}^2 \tag{33a}$$

$$\rho_1^p = \frac{1 - \gamma^r + \gamma^e}{1 + \gamma^e} \tag{33b}$$

$$\rho_2^p = \frac{(1 - \gamma^r)^2 + \gamma^e (1 - 2\gamma^r)}{1 + \gamma^e} \tag{33c}$$

$$\rho_i^p = (1 - \gamma^r + \gamma^e)\rho_{i-1}^p - \gamma^e \rho_{i-2}^p \ , \text{ for i} \geq 3. \tag{33d}$$

We note that the serial correlation pattern is determined by the proportions of the rational and emotional investors n^r and n^e, by the characteristic parameters of the rational and emotional beliefs

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formation c^r, c^e, k^e, a, b, as well as by the market liquidity and the emotional demand sensitivity β^e.

3. rational strategy following Kyle (1985): $\tilde{r}^r_{t-1} = -p_{t-1}$ and emotional trend-following strategy: $\tilde{r}^e_{t-1} = r_{t-1} - r_{t-2}$

Using the same notations as above, log-prices reduce to

$$p_t \quad = \gamma + (1 - \gamma^r + \gamma^e)p_{t-1} - 2\gamma^e p_{t-2} + \gamma^e p_{t-3} + \xi_t. \tag{34}$$

The simulations from Section 3 for the cases 1 and 2 (with low and medium proportion of emotional traders active in the market) yield mostly to stationary log-price and log-returns series. Only in case 2c, the first log-price serial correlation exhibits a value close to 0.05. However, the price serial correlations decrease drastically for higher lags.

Mostly, the cross-correlations between log-returns and lagged rational and emotional expectations exhibit similar magnitudes and patterns. This supports the idea that the emotional strategy is comparably good with respect to the rational one in order to correctly assess information about future prices.

6 Appendix B
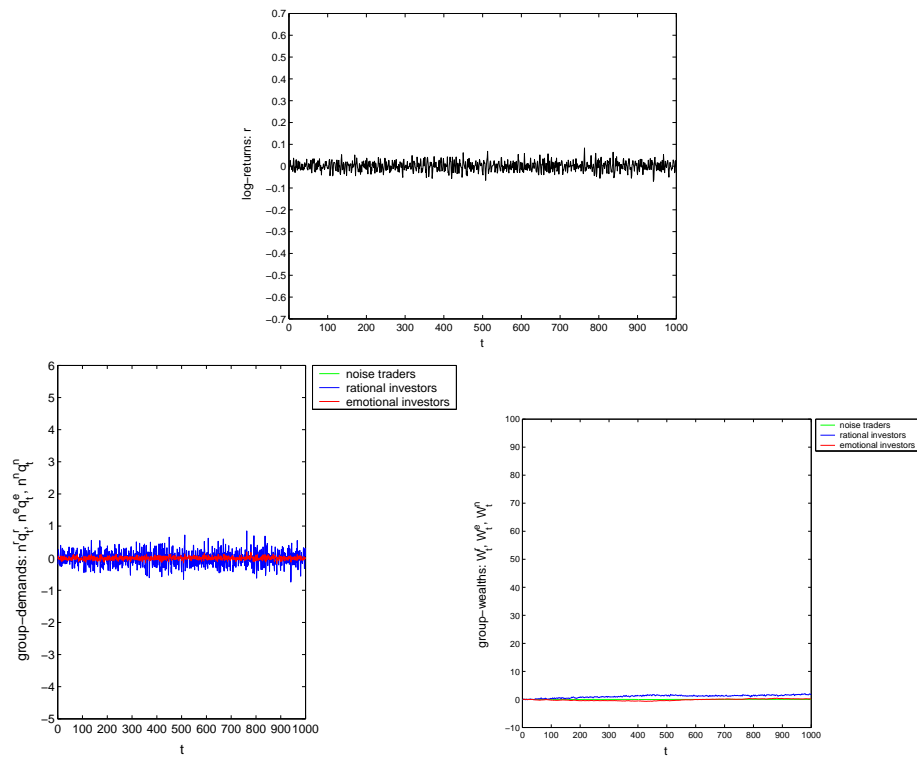
Figure 1: Log-returns, group-demands and group-wealths in case 1a: $n^e = 0.25$, $a = b = 1$
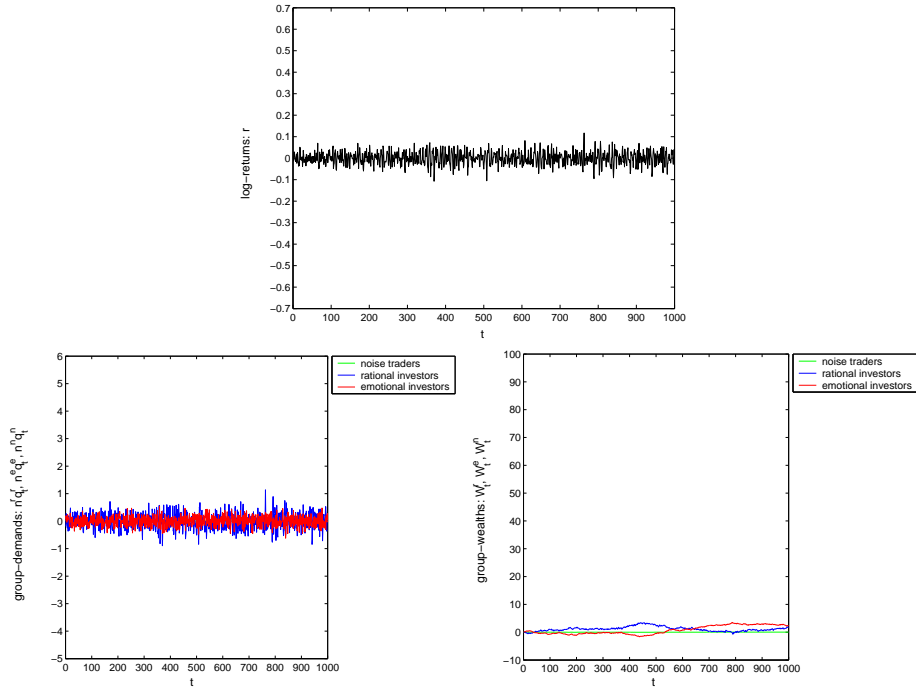
Figure 2: Log-returns, group-demands and group-wealths in case 1b: $n^e = 0.25$, $a = 0.01$, $b = 5$
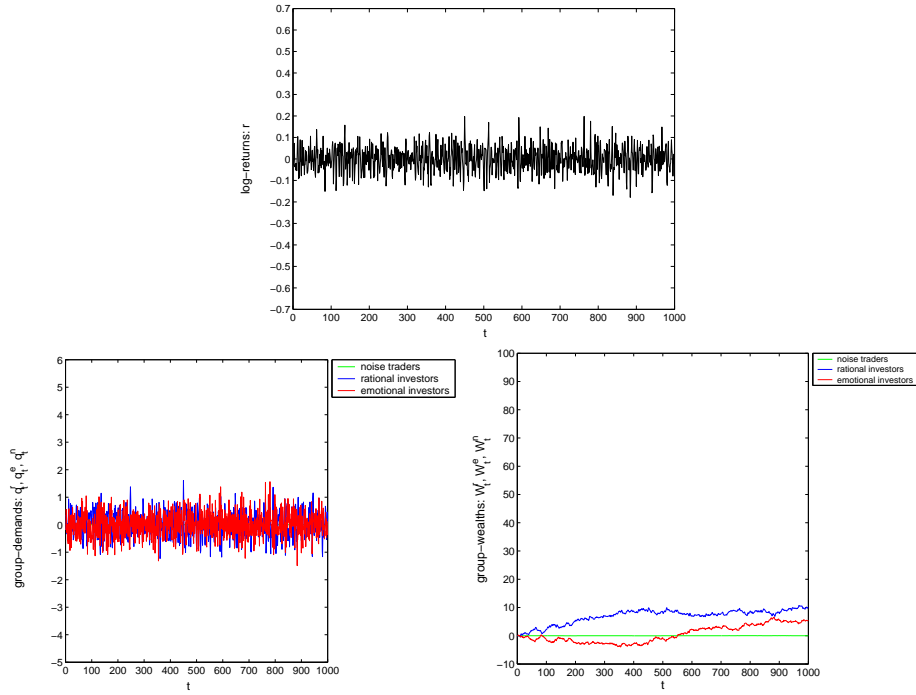

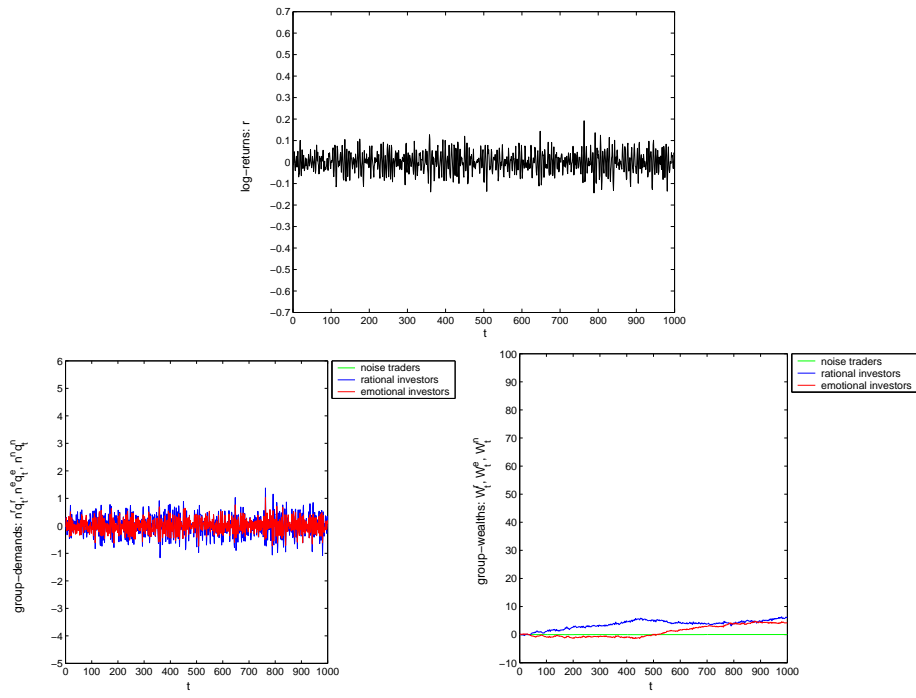Figure 3: Log-returns, group-demands and group-wealths in case 1c: $n^e = 0.25$, $a = 5$, $b = 0.01$

Figure 4: Log-returns, group-demands and group-wealths in case 2a: $n^e = 0.5$, $a = b = 1$
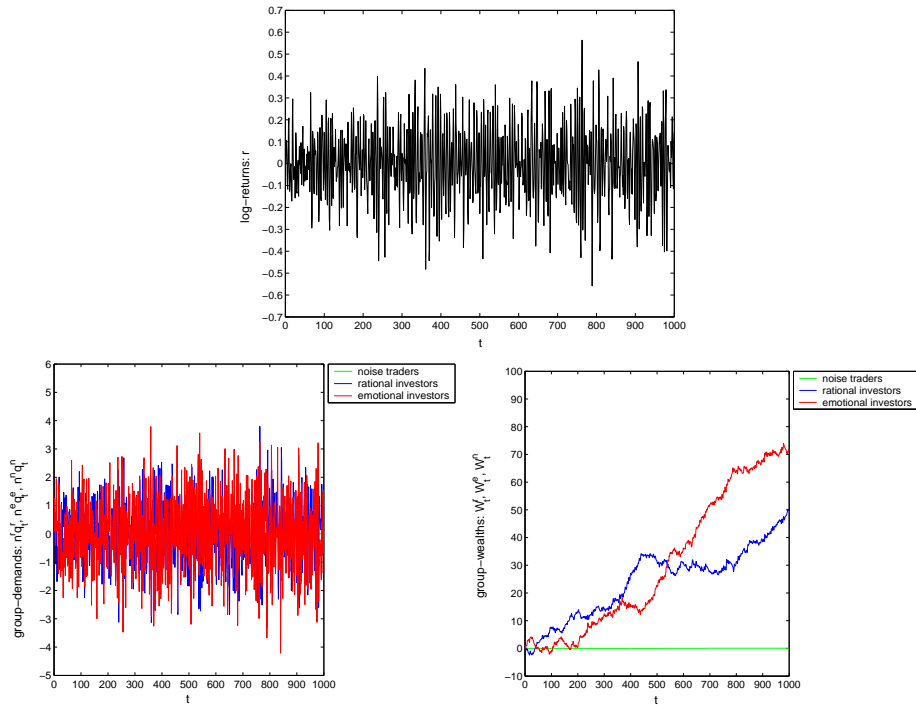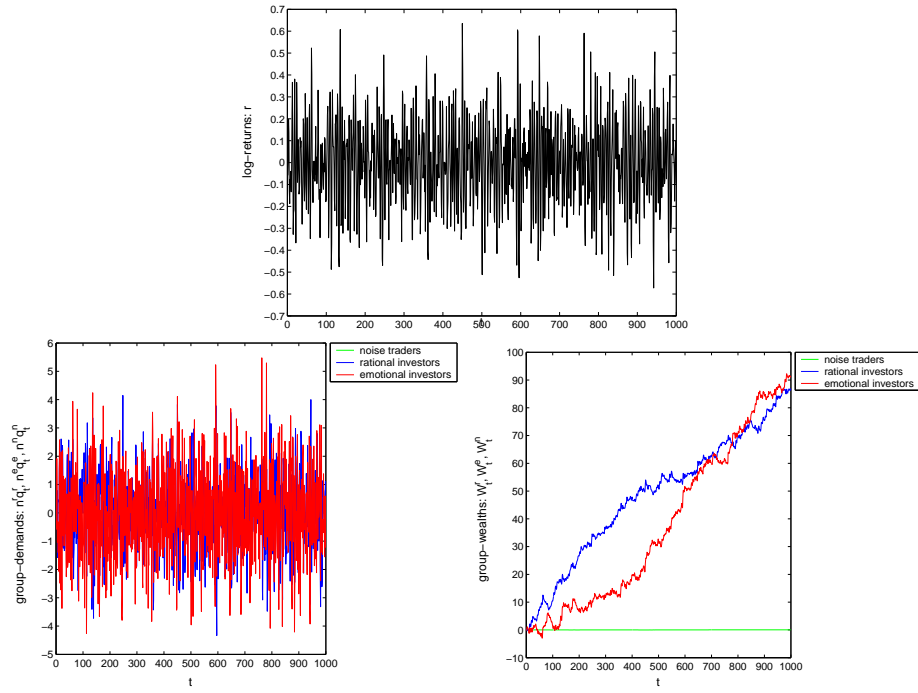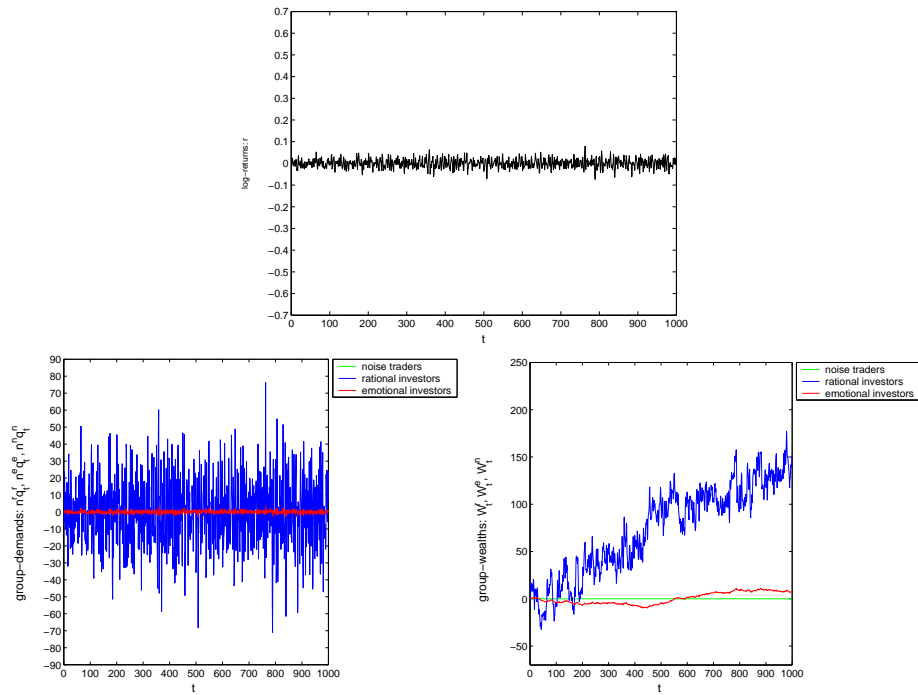

Figure 5: Log-returns, group-demands and group-wealths in case 2b: $n^e = 0.25$, $a = 0.01$, $b = 5$

Figure 6: Log-returns, group-demands and group-wealths in case 2c: $n^e = 0.25$, $a = 5$, $b = 0.01$

Figure 7: Log-returns, group-demands and group-wealths in case 3a: $n^e = 0.25$, $a = b = 1$, for a more liquid market: $\lambda = 0.001$
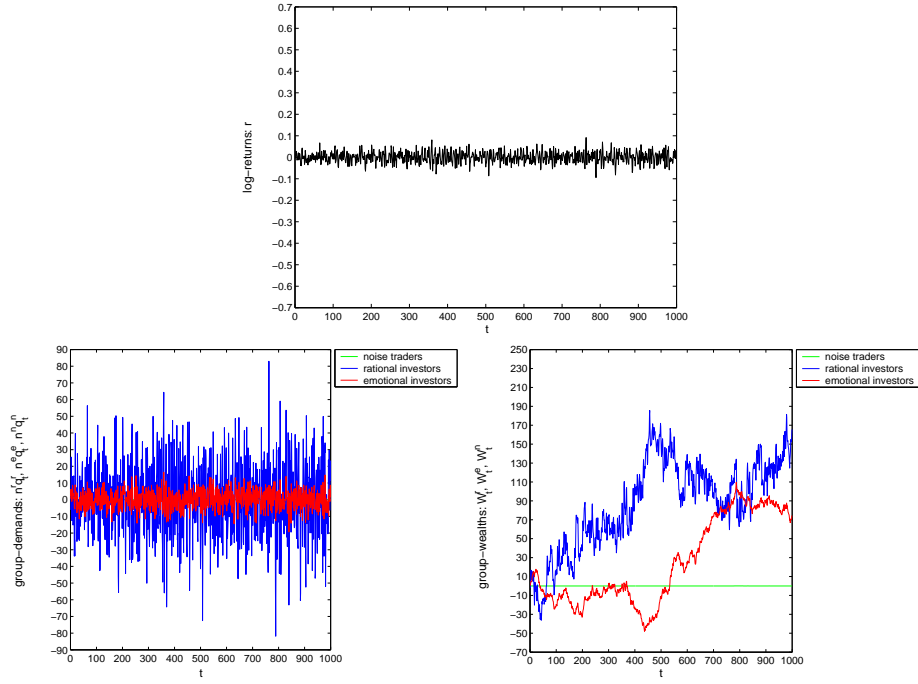
Figure 8: Log-returns, group-demands and group-wealths in case 3b: $n^e = 0.25$, $a = 0.01$, $b = 5$ for a more liquid market: $\lambda = 0.001$
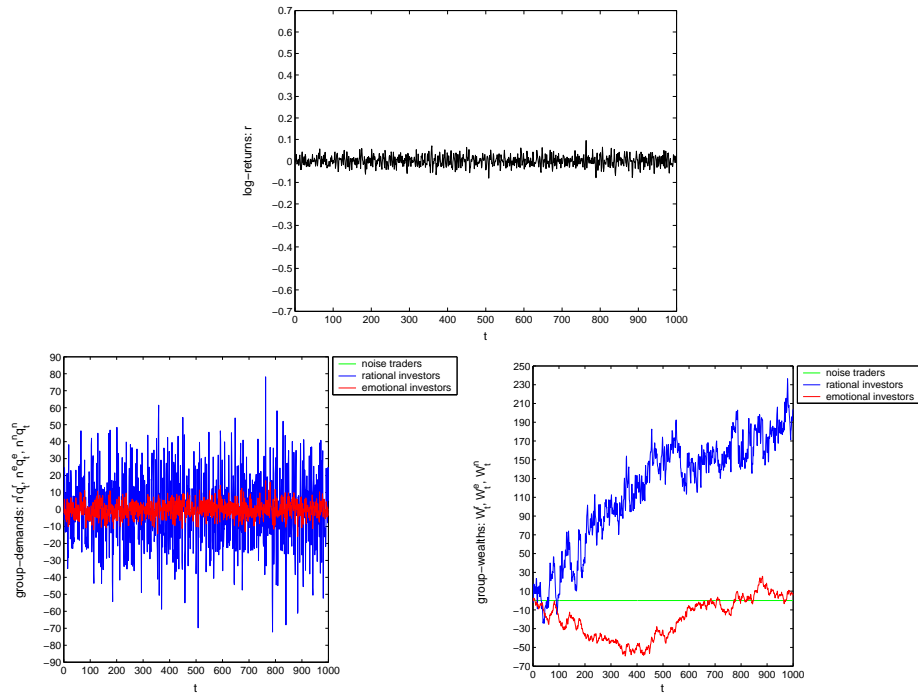

Figure 9: Log-returns, group-demands and group-wealths in case 3c: $n^e = 0.25$, $a = 5$, $b = 0.01$ for a more liquid market: $\lambda = 0.001$